LEMBAR KEGIATAN PESERTA DIDIK

MATRIKS

PERTEMUAN II

Nama	
Kelas	

Kompetensi Dasar:

3.16. Menentukan nilai determinan, invers dan tranpos pada ordo 2x2 dan nilai determinan dan tranpos pada ordo 3x3

4.16 Menyelesaikan masalah yang berkaitan dengan determinan, invers dan tranpose pada ordo 2x2 serta nilai determinan dan tranpos pada ordo 3x3

Indikator Pencapaian Kompetensi

3.16.1 Menentukan transpose matriks
3.16.2 Menentukan nilai determinan matriks ordo 2x2
3.16.3 Menentukan invers matriks ordo 2x2
3.16.4 Menentukan determinan matriks ordo 3x3
4.16.1 Menyelesaikan masalah menggunakan determinan dan invers matriks ordo 2x2
4.16.2 Menyelesaikan masalah dengan menentukan determinan matriks ordo 3x3

Tujuan Pembelajaran

Melalui model pembelajaran discovery learning dan aplikasi *Google meet, Google Classroom* serta WA Grup, peserta didik dapat menentukan transpose matriks, menentukan nilai determinan dan invers matriks ordo 2 x 2 dan menentukan determinan matriks ordo 3x3 serta memiliki sikap disiplin dan kerjasama.

PETUNJUK PENGGUNAAN LKPD

1. LKPD ini dapat di Download melalui Google Classroom
2. Bacalah LKPD ini dengan cermat.
3. Diskusikanlah LKPD ini dengan teman sekelompokmu.
4. Tanyakan pada guru apabila mendapat kesulitan dalam mengerjakan LKPD.
5. Tuliskan jawabanmu pada LKPD ini.
6. Setelah selesai mengerjakan LKPD, setiap kelompok akan mempresentasikan melalui Google meet pada saat Jadwal Pembelajaran Daring.

A • Invers Matriks

Def. Jika matriks A.B = B.A = I maka kedua matriks itu saling invers ditulis A = B $^{-1}$ dan B = A $^{-1}$
maka A.A $^{-1}$ = A $^{-1}$.A = I

Rumus Invers matriks berordo 2x2

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} .... & .... \\ .... & .... \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = ...\ .\begin{pmatrix} ..... & ...... \\ ..... & ...... \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} .... \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} ..... & ...... \\ ..... & ...... \end{pmatrix}$$

Jadi bila $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ maka $A^{-1} = \dfrac{..}{...............} . \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix}$

dengan $a.d - b.c$ disebut determinan matriks A ditulis $\det A = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = a.d - b.c$

dan invers matriks A bisa juga ditulis $A^{-1} = \frac{..}{..............} . \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix}$

Apakah setiap matriks mempunyai invers ? (…………………)
Matriks **P** yang tidak punya invers disebut matriks ***singular*** bila ***det P*** **= ….**

Contoh :

1. Tentukan determinan matriks berikut ini :

a. $A = \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix} \Rightarrow \det A = \begin{vmatrix} 3 & 5 \\ -2 & 4 \end{vmatrix} = (3).(4) - (5).(-2) = ...-.......= .....$

b. $B = \begin{pmatrix} -1 & 4 \\ 5 & 6 \end{pmatrix} \Rightarrow \det B = \begin{vmatrix} -1 & 4 \\ 5 & 6 \end{vmatrix} = (....).(....) - (....).(....) = ...-.......= .....$

c. $P = \begin{pmatrix} -6 & 7 \\ 2 & -3 \end{pmatrix} \Rightarrow \det P = \begin{vmatrix} ... & ... \\ ... & ... \end{vmatrix} = (....).(....) - (....).(....) = ...-.......= .....$

d. $Q = \begin{pmatrix} -4 & 2 \\ -5 & 3 \end{pmatrix} \Rightarrow \det Q = \begin{vmatrix} ... & ... \\ ... & ... \end{vmatrix} = (....).(....) - (....).(....) = ...-.......= .....$

2. Manakah diantara matriks berikut yang singular?

a. $\begin{pmatrix} 3 & 6 \\ -2 & 4 \end{pmatrix}$ b. $\begin{pmatrix} -2 & -6 \\ 3 & 9 \end{pmatrix}$ c. $\begin{pmatrix} -10 & 5 \\ 4 & -2 \end{pmatrix}$ d. $\begin{pmatrix} -3 & 6 \\ -4 & -8 \end{pmatrix}$

3. Diketahui matriks $A = \begin{pmatrix} -3 & x \\ 2 & 4 \end{pmatrix}$, tentukan nilai x bila matriks A singular.

Matriks A singular maka $\det A = 0$

$$(-3).(4) - (x).(2) = 0$$
$$...... - ..... = 0$$
$$...... = ...$$
$$x = ...$$

4. Diketahui matriks $P = \begin{pmatrix} 2x & 4x \\ 3 & x \end{pmatrix}$, tentukan nilai x bila matriks P singular.

Matriks P singular maka $\det P = 0$

$$(.....).(.....) - (.....).(.....) = 0$$
$$........ - .... = 0$$
$$...... (............) = 0$$
$$x = ... \quad atau \quad x = ...$$

5. Tentukan invers dari matriks berikut!

a. $P = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$ maka $P^{-1} = \frac{1}{.............} \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix} = \frac{1}{....} \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix} = \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix}$

b. $Q = \begin{pmatrix} -7 & 4 \\ -5 & 3 \end{pmatrix}$ maka $Q^{-1} = \frac{1}{..........} \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix} = \frac{1}{....} \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix} = \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix}$

c. $K = \begin{pmatrix} 5 & -6 \\ -2 & 3 \end{pmatrix}$ maka $K^{-1} = \frac{1}{..........} \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix} = \frac{1}{....} \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix} = \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix}$

d. $L = \begin{pmatrix} 8 & -7 \\ 4 & -3 \end{pmatrix}$ maka $L^{-1} = \frac{1}{.............} \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix} = \frac{1}{....} \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix} = \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix}$

e. $M = \begin{pmatrix} -9 & 4 \\ 5 & -2 \end{pmatrix}$ maka $M^{-1} = \frac{1}{.............} \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix} = \frac{1}{....} \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix} = \begin{pmatrix} ..... & ..... \\ ..... & ..... \end{pmatrix}$

B •Sifat-sifat Invers matriks berordo 2x2

Diketahui matriks : $A = \begin{pmatrix} -2 & 4 \\ 3 & -5 \end{pmatrix}$ dan $B = \begin{pmatrix} -6 & 3 \\ -5 & 2 \end{pmatrix}$

Hitunglah :

1. $A^{-1} = \frac{1}{\dots\dots\dots\dots} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

2. $B^{-1} = \frac{1}{\dots\dots\dots\dots} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

3. $A.A^{-1} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

4. $A^{-1}.A = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

5. $B.B^{-1} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

6. $B^{-1}.B = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

7. $A.B = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

8. $B.A = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

9. $(A.B)^{-1} = \frac{1}{\dots\dots\dots\dots} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

10. $(B.A)^{-1} = \frac{1}{\dots\dots\dots\dots} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

11. $A^{-1}.B^{-1} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

12. $B^{-1}.A^{-1} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix} = \begin{pmatrix} \dots\dots & \dots\dots \\ \dots\dots & \dots\dots \end{pmatrix}$

Jadi bila diketahui matriks A dan B maka sifat-sifat berikut berlaku :

(i) $A.A^{-1} = A^{-1}.A = \dots\dots$ (……………………………..)

$$\left.\begin{array}{ll} \text{(ii)} & (A.B)^{-1} = \dots\dots\dots\dots \\ \text{(iii)} & (B.A)^{-1} = \dots\dots\dots\dots \end{array}\right\} (\textit{sifat} \ \dots\dots\dots\dots\dots\dots\dots\dots)$$

C • Mengalikan dari Kiri dan Mengalikan dari Kanan

Mengalikan dari Kiri

$$A.X = B$$
$$A^{-1}.A.X = A^{-1}.B$$
$$I.X = A^{-1}.B$$
$$X = A^{-1}.B$$

Mengalikan dari Kanan

$$X.A = B$$
$$X.A. \ldots. = B. \ldots..$$
$$X. \ldots.. = B. \ldots..$$
$$X = B. \ldots..$$

Contoh : Tentukan Matriks X yang memnuhi persamaan berikut.

1. $\begin{pmatrix} 2 & -2 \\ 5 & -4 \end{pmatrix}.X = \begin{pmatrix} -4 & 6 \\ -11 & 17 \end{pmatrix}$ maka $X = \begin{pmatrix} 2 & -2 \\ 5 & -4 \end{pmatrix}^{-1}\begin{pmatrix} -4 & 6 \\ -11 & 17 \end{pmatrix}$

$$X = \frac{1}{\ldots\ldots\ldots\ldots\ldots}\begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}\begin{pmatrix} -4 & 6 \\ -11 & 17 \end{pmatrix}$$
$$X = \frac{1}{\ldots\ldots}\begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}$$
$$X = \begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}$$

2. $\begin{pmatrix} 7 & -2 \\ -9 & 3 \end{pmatrix}.X = \begin{pmatrix} 23 \\ -27 \end{pmatrix}$ maka $X = \begin{pmatrix} 7 & -2 \\ -9 & 3 \end{pmatrix}^{-1}\begin{pmatrix} 23 \\ -27 \end{pmatrix}$

$$X = \frac{\ldots\ldots}{\ldots\ldots\ldots\ldots\ldots}\begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}\begin{pmatrix} 23 \\ -27 \end{pmatrix}$$
$$X = \frac{\ldots\ldots}{\ldots..}\begin{pmatrix} \ldots\ldots. \\ \ldots\ldots. \end{pmatrix}$$
$$X = \begin{pmatrix} \ldots\ldots. \\ \ldots\ldots. \end{pmatrix}$$

3. $X.\begin{pmatrix} -5 & 2 \\ 8 & -4 \end{pmatrix} = \begin{pmatrix} 12 & -8 \\ -18 & 4 \end{pmatrix}$ maka $X = \begin{pmatrix} 12 & -8 \\ -18 & 4 \end{pmatrix}\begin{pmatrix} -5 & 2 \\ 8 & -4 \end{pmatrix}^{-1}$

$$X = \begin{pmatrix} 12 & -8 \\ -18 & 4 \end{pmatrix}\frac{\ldots..}{\ldots\ldots\ldots\ldots}\begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}$$
$$X = \frac{1}{\ldots\ldots}\begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}$$
$$X = \begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}$$

4. $X.\begin{pmatrix} -3 & 4 \\ -4 & 7 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 0 & 5 \end{pmatrix}$ maka $X = \begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}\begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}^{\ldots..}$

$$X = \begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}\frac{\ldots..}{\ldots\ldots\ldots\ldots}\begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}$$
$$X = \frac{1}{\ldots\ldots}\begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}$$
$$X = \begin{pmatrix} \ldots.. & \ldots.. \\ \ldots.. & \ldots.. \end{pmatrix}$$

5. Diketahui matriks $A = \begin{pmatrix} 3 & -5 \\ 2 & -4 \end{pmatrix}$ *dan* $B = \begin{pmatrix} 9 & 17 \\ 8 & 12 \end{pmatrix}$, tentukan matriks X_{2x2} yang memenuhi persamaan $A.X = B$

D **•Pemakaian Matriks dalam Menyelesaikan SPL**

(i) Dengan menggunakan invers matriks

Contoh : Tentukan penyelesaian sistem persamaan linear berikut ini :

1. $\left.\begin{matrix} 2x - 3y = 11 \\ 5x + y \ = 19 \end{matrix}\right\}$ dapat diubah menjadi matriks $\begin{pmatrix} 2 & -3 \\ 5 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 19 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ 5 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 11 \\ 19 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\ldots\ldots\ldots\ldots}\begin{pmatrix} \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots \end{pmatrix}\begin{pmatrix} 11 \\ 19 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\ldots\ldots}\begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$$

Jadi $x = \ldots.$ *dan* $y = \ldots.$

2. $\left.\begin{matrix} 4x + 3y = 0 \\ 7x - 2y = -29 \end{matrix}\right\}$ dapat diubah menjadi matriks $\begin{pmatrix} \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots \end{pmatrix}^{-1}\begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\ldots\ldots\ldots\ldots}\begin{pmatrix} \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots \end{pmatrix}\begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\ldots\ldots}\begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\ldots\ldots}\begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$$

Jadi $x = \ldots.$ *dan* $y = \ldots.$

3. $\left.\begin{matrix} -3x + 5y = -5 \\ 4x - 7y = 6 \end{matrix}\right\}$

4. $\left.\begin{matrix} -x - 2y = -3 \\ 2x + 3y = -2 \end{matrix}\right\}$

(ii) Dengan menggunakan determinan matriks

Diketahui sistem persamaan linear $\left.\begin{array}{l} ax + by = c \\ px + qy = r \end{array}\right\}$

Maka $\Delta = \begin{vmatrix} a & b \\ p & q \end{vmatrix}$; $\Delta x = \begin{vmatrix} c & b \\ p & r \end{vmatrix}$ dan $\Delta y = \begin{vmatrix} a & c \\ p & r \end{vmatrix}$

Sehingga $x = \dfrac{\Delta x}{\Delta}$ dan $y = \dfrac{\Delta y}{\Delta}$

Contoh : Tentukan penyelesaian sistem persamaan linear berikut ini :

1. $\left.\begin{array}{l} 2x - 3y = 11 \\ 5x + y \ \ = 19 \end{array}\right\}$ diperoleh $\Delta = \begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = (2).(1) - (-3)(5) = ... - ..... = ...$

$$\Delta x = \begin{vmatrix} 11 & -3 \\ 19 & 1 \end{vmatrix} = (11).(1) - (-3)(19) = ... - ..... = ...$$

$$\Delta y = \begin{vmatrix} 2 & 11 \\ 5 & 19 \end{vmatrix} = (2).(19) - (11)(5) = ... - ..... = ...$$

Jadi $x = \dfrac{\Delta x}{\Delta} = \dfrac{.....}{.....} = ...$ dan $y = \dfrac{\Delta y}{\Delta} = \dfrac{.....}{.....} = ...$

2. $\left.\begin{array}{l} -x - 2y = -3 \\ 2x + 3y = -2 \end{array}\right\}$ diperoleh $\Delta = \begin{vmatrix} ..... & ..... \\ ..... & ..... \end{vmatrix} = (.....).(.....) - (.....)(.....) = ... - ..... = ...$

$$\Delta x = \begin{vmatrix} ..... & ..... \\ ..... & ..... \end{vmatrix} = (.....).(.....) - (.....)(.....) = ... - ..... = ...$$

$$\Delta y = \begin{vmatrix} ..... & ..... \\ ..... & ..... \end{vmatrix} = (.....).(.....) - (.....)(.....) = ... - ..... = ...$$

Jadi $x = \dfrac{\Delta x}{\Delta} = \dfrac{.....}{.....} = ...$ dan $y = \dfrac{\Delta y}{\Delta} = \dfrac{.....}{.....} = ...$

3. $\left.\begin{array}{l} 4x + 3y = -6 \\ 5x - 2y = 27 \end{array}\right\}$

4. $\left.\begin{array}{l} -x + 7y \ = 17 \\ \ \ 5x - 2y = 14 \end{array}\right\}$

D •Menentukan determinan matriks persegi berordo 3x3

Menentukan determinan matriks $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Ada dua cara menentukan determinan matriks persegi berordo 3x3 yaitu :

b. Cara Sarrus

Determinan matriks A = $\det A = \left| \begin{array}{ccc|cc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array} \right.$

$$= (a_{11}.a_{22}.a_{33} + a_{12}.a_{23}.a_{31} + a_{13}.a_{21}.a_{32}) - (a_{31}.a_{22}.a_{13} + a_{32}.a_{23}.a_{11} + a_{33}.a_{21}.a_{12})$$

c. Kaidah Cramer

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Contoh :

1. Hitunglah determinan matriks $A = \begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{bmatrix}$

Jawab :

Cara Sarrus

$$\det A = \left| \begin{array}{ccc|cc} 3 & 1 & 0 & \ldots & \ldots \\ -2 & -4 & 3 & \ldots & \ldots \\ 5 & 4 & -2 & \ldots & \ldots \end{array} \right. = (\ldots \quad \ldots \quad \ldots) - (\ldots \quad \ldots \quad \ldots) = \ldots \quad \ldots = \ldots.$$

Cara Cramer

$$\det A = \begin{vmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{vmatrix} = \ldots \begin{vmatrix} \ldots & \ldots \\ \ldots & \ldots \end{vmatrix} \quad \ldots \begin{vmatrix} \ldots & \ldots \\ \ldots & \ldots \end{vmatrix} \quad \ldots \begin{vmatrix} \ldots & \ldots \\ \ldots & \ldots \end{vmatrix} = \ldots(\ldots) \quad \ldots(\ldots) \quad \ldots(\ldots) = \ldots..$$

2. Hitunglah determinan matriks $B = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -2 & 4 \\ -3 & 1 & -5 \end{bmatrix}$

$$\det B = \left|\begin{array}{ccc|cc} \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{array}\right| = (\ldots \quad \ldots \quad \ldots) - (\ldots \quad \ldots \quad \ldots) = \ldots \quad \ldots = \ldots.$$

3. Hitunglah determinan matriks $C = \begin{bmatrix} 2 & -1 & 4 \\ -4 & -2 & 3 \\ 1 & -3 & -1 \end{bmatrix}$

$$\det C = \begin{vmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{vmatrix} = \ldots \begin{vmatrix} \ldots & \ldots \\ \ldots & \ldots \end{vmatrix} \quad \ldots \begin{vmatrix} \ldots & \ldots \\ \ldots & \ldots \end{vmatrix} \quad \ldots \begin{vmatrix} \ldots & \ldots \\ \ldots & \ldots \end{vmatrix} = \ldots(\ldots) \quad \ldots(\ldots) \quad \ldots(\ldots) = \ldots..$$

4. Tentukan penyelesaian sistem persamaan linear berikut ini :

a. $\left.\begin{array}{l} 2x + y - 3z = -1 \\ 3x - 2y + z = 13 \\ x - 3y - 2z = 2 \end{array}\right\}$

Jawab :

$$\Delta = \left|\begin{array}{ccc|cc} 2 & 1 & -3 & 2 & 1 \\ 3 & -2 & 1 & 3 & -2 \\ 1 & -3 & -2 & 1 & -3 \end{array}\right| = (8 + 1 + 27) - (6 - 6 - 6) = 36 - (-6) = 42$$

$$\Delta_x = \left|\begin{array}{ccc|cc} -1 & 1 & -3 & -1 & 1 \\ 13 & -2 & 1 & 13 & -2 \\ 2 & -3 & -2 & 2 & -3 \end{array}\right| = (-4 + 2 + 117) - (12 + 3 - 26) = 115 - (-11) = 126$$

$$\Delta_y = \left|\begin{array}{ccc|cc} 2 & -1 & -3 & 2 & -1 \\ 3 & 13 & 1 & 3 & 13 \\ 1 & 2 & -2 & 1 & 2 \end{array}\right| = (-52 - 1 - 18) - (-39 + 4 + 6) = -71 - (-29) = -42$$

$$\Delta_z = \left|\begin{array}{ccc|cc} 2 & 1 & -1 & 2 & 1 \\ 3 & -2 & 13 & 3 & -2 \\ 1 & -3 & 2 & 1 & -3 \end{array}\right| = (-8 + 13 + 9) - (2 - 78 + 6) = 14 - (-70) = 84$$

$$x = \frac{126}{42} = 3 \ ; \quad y = \frac{-42}{42} = -1 \quad \text{dan} \quad z = \frac{84}{42} = 2$$

b. $\left.\begin{array}{l} 3x - 2y - z = -15 \\ 4x - y + 3z = -16 \\ x + 3y + 5z = 2 \end{array}\right\}$